Math 153 (Revised: 2/18/17)
Foundations: Number Systems

Advising

This core course is a **requirement** for **Early Childhood, Elementary Education**, and **Special Education majors** and should be taken early in their program (first or second year).

This is a mathematics subject matter course which covers the covers the number systems strand of the Massachusetts Curriculum Frameworks in Mathematics at a collegiate level. To meet Massachusetts teacher licensure requirements, students are required to take Math 153, MATH 250, and a math methods course (EDUC 201 or MATH 356). To have comprehensive coverage of the content strands in Mathematics, we strongly recommend that students take Math 153, 250, 251, and Math 252.

Math 153 is **inappropriate** for students who are not interested in Elementary Mathematics Education. Students who are just looking to satisfy the Mathematics Core Requirement and have not specific requirement or recommendation for their major, **Math 110 and/or Math 111 may be more appropriate choices.**

Course Description

An introductory course. Topics include: inductive and deductive reasoning; problem solving; logic; the development and properties of various number systems (such as integers, rational, real and complex numbers), as well as operations and different representations in these number systems (e.g. in bases other than 10). Students will develop a conceptual understanding of the course material in a learning environment that models the pedagogical foundations of the Massachusetts Curriculum Frameworks for Mathematics and the National Council of Teachers of Mathematics (NCTM) Standards.

Pedagogy

Part of the course objective is to introduce students to some of the teaching pedagogy outlined by the NCTM.
Following the NCTM Standards, the course is designed to educate students to become *active* participants (rather than passive observers) in mathematical thinking, and to encourage them to educate their future students in the same spirit. This may be done using some or all of the following approaches:

- *Small group work*
- *Emphasis on student verbal explanation* of problem-solving processes rather than just providing answers. Verbally explaining or defending solutions can help develop students' mathematical thinking, as well as articulative skill.
- *Increasing students' self-reliance* on checking solutions leads to deeper mathematical thinking. They have to think about whether or not their answers make sense; they have to think about the problem further to devise a way to check the solution; and their understanding and thinking about the problem will be deepened. If the professor simply says, "That's right" or "That's wrong," thinking about the problem will immediately cease.

- *The Constructivist approach* to learning is emphasized: students discover and build mathematical concepts themselves, rather than just memorizing them without really understanding. According to schema learning theory, knowledge is acquired in greater depth and is more efficiently retained if it can be connected with the learner's pre-existing knowledge. It cannot be assumed that learners will make the connections without help. The connections of new material to other knowledge structures must be made; and in keeping with the self-reliance issue raised earlier, the connections should be made by the students themselves whenever possible.
- *Using manipulatives* as a tool for understanding mathematical concepts and for solving problems.
- *Developing different problem-solving strategies* (estimating, drawing a diagram, discovering patterns, constructing a table, etc.) that are applicable to a wide variety of situations.

Other objectives vary from instructor to instructor. There is a real concern that many students preparing to be elementary school teachers lack basic arithmetic skills that they need to teach their students. This issue is frequently addressed in the "plus" sections of MA 150. Another concern is that the attitude they as teachers have about mathematics is likely to carry over into their own classroom; keeping a journal is one way of helping students deal with this issue.

Course Objectives:

Upon completion of this course students will have learned:

1. Problem solving techniques
2. Inductive and deductive reasoning
3. Logic
4. The conceptual, mathematical, and historical development of several important numbers systems including several of the following: natural numbers, integers, rational numbers, real numbers, complex numbers, transfinite ordinal numbers, transfinite cardinal numbers, and modular number fields
5. Different representations of numbers in these number systems, including: base 10 numeration, representations in bases other than 10, fractions, decimals, and scientific notation
6. Operations in these number systems, including: addition, subtraction, multiplication, division, and exponentiation
7. About key properties in these number systems, including: divisibility and factorization, associativity, commutativity, distributivity, closure, and density

Instructional Objectives

Upon completion of the course, students will be able to:

- Appropriately select and apply different problem-solving techniques
- Clearly articulate the problem-solving method(s) used
- Utilize mathematical and logical reasoning
- Construct simple proofs
- Describe the difference between deductive and inductive reasoning, including the strengths and limits of each

Resources

- Bassarear, T. Mathematics for Elementary School Teachers, 3rd Ed., Houghton-Mifflin Co.
- Bennett and Nelson. Mathematics for Elementary Teachers, 6th Ed., McGraw Hill Publishing; ISBN: 0-072-53298-X + supplements & manipulative kit.
- Van De Walle. Elementary and Middle School Mathematics: Teaching Developmentally, *x*th Ed., Pearson, 20xx.

It is strongly recommended that anyone teaching the course for the first time read one or more of the following three books:

- Boaler, Jo. What's Math Got To Do With It?: How Teachers and Parents Can Transform Mathematics Learning and Inspire Success, Penguin Books; Revised ed. edition, March 31, 2015.
- Ma, Liping. Knowing and Teaching Elementary Mathematics, Lawrence Erlbaum Associates, Inc., 2010.
- Ohanian, S. Garbage Pizza, Patchwork Quilts, and Math Magic, W. H. Freeman and Co., 1994.
- Schifter, Deborah (ed.). Reconstructing Mathematics Education, Teachers College Press, 1993. *(May be out of print, department has a few extra copies to loan out.)*

The following videotape resources are available from the math department:

- Young Mathematicians At Work (about 10 CDs).
- Developing Mathematical Ideas (DMI), (Videotapes and DVDs).
- Teaching Math: a Video Library, K-4, Annenberg/CPB Mathematical Science Collection, WGBH, 1995. (A collection of 26 tapes and 11 DVDs)
- Challenge in the Classroom, Mathematical Association of America (no date given).
- Mathematics: Making the Connection, NCTM, 1991.
- Discovery Workshop, NCTM, 1991.
- Using Numbers: Real Data in the Classroom, Dale Seymour Productions, 1990.
- Learning Games, Frog Publications, 1997.